PULSAR MODELS

W.J. Cocke*

Jeffrey M. Cohen*

Institute for Space Studies
Goddard Space Flight Center, NASA
New York 10025

In this paper we examine the possibility, considered previously by others, that the recently observed pulsed radio sources are pulsating white dwarfs. We first present the results of radial mode eigenvalue calculations (for various central densities and compositions) which were obtained by integrating the eigenvalue equations themselves rather than by using variational techniques.

Computation of non-linear white dwarf pulsations are then presented, in which a hydrodynamic code was used to follow finite amplitude pulsations driven by pycnonuclear reactions. Very large amplitude pulsation of neutron stars are also considered.

The possibility that the optical pulsation periods are twice as long as the radio periods has arisen from observations.[1] We discuss a method for such a "frequency doubling" based partly on magnetohydrodynamics.

Eigenvalue Calculations

The following white dwarf eigenfrequency calculations were obtained by computer integration of the Newtonian equations for the eigenfunctions. The eigenvalues were obtained by finding those values of the period for which the eigenfunction $\xi = \delta r/r$ and $d\xi/dr$ are finite at the outer boundary and match to an analytic expansion (about the surface) of the eigenfunction. The eigenvalues compare very well with those determined by variational methods[2], but the method described here has the advantage of giving accurate eigenfunctions without having to postulate polynomial forms. In table 1 we have used the degenerate electron equation of state as first applied to white dwarfs by Chandrasekhar[3]. The effect of Hamada-Salpeter corrections to the equation of state[4], etc., is found to reduce the periods slightly. In table 2 we show the results of calculations corrected for the Coulomb Thomas-Fermi and exchange interactions (Hamada-Salpeter corrections). In both tables the ratio of nucleons to electrons was taken to be 2.0. Thus the un -corrected models can represent for example C^{12} or C^{12} + He^4 compositions while the corrected model assumes $Z = 6$ as with C^{12}.

For higher densities than those considered here, we have found that relativistic effects are too large to neglect[5], and that periods considerably lower than 1.64 sec are very difficult to obtain. This difficulty, however, can be circumvented. We later give a mechanism which generates two radio frequency pulses for each stellar pulsation. Also, in the following sections we discuss dynamical calculations with simulated nuclear excitations, which tend to show that pulsation periods shorter than the fundamental may in any case be obtained. Thus there are at least two possible ways of producing models with radio periods shorter than the fundamental periods.

Dynamical Calculations

The full, non-linear hydro-dynamical equations of motion were used to follow the oscillations of a white dwarf starting from initial conditions close to equilibrium. We employ the Lagrangian form[6]

$$\frac{\partial^2 r}{\partial t^2} = -4\pi r^2 \partial_m (P+Q) - \frac{GM}{r^2}$$

where m is the mass interior to radius $r = r(m,t)$, p is the pressure, and Q is a viscosity term introduced both to spread out shock fronts over several mass zones, and to eliminate the effects of the initial conditions on the final state of motion of the white dwarf. We employed 20 mass zones of equal mass, and used a form for Q given by Colgate and White[7] and Arnett[8] as $Q_{k+\frac{1}{2}} = C\rho_{k+\frac{1}{2}}(v_{k+\frac{1}{2}}-v_k)^2$ for $v_{k+1} < v_k$ and $Q_{k+\frac{1}{2}} = 0$ otherwise, where v_k is the velocity of k-th mass shell, $\rho_{k+\frac{1}{2}}$ is the density, and C is a constant. Arnett[8] chooses C = 1, but we chose C = 10 in order to damp out the effects of the initial conditions in reasonable computer time.

The pressure is given by two terms $P = P_e + P_n$, where P_e is the usual degenerate electron pressure and P_n is due to pycnonuclear reactions[9]. We have used the form $P_n \propto (\rho - \rho_1)^2$, where ρ_1 is a density at which the excess pressure from the heating from the nuclear reactions was assumed to die away. The reactions were assumed to cut on at a density $\rho_2 > \rho_1$. The total mass in the model used was assumed to be $M = 8.38 \times 10^{32}$g, corresponding to a central density ρ_c of about 2×10^6g/cm^3. Of course, no pycnonuclear reactions actually occur at this density. We would have liked to have run models with densities high enough for these reactions to occur, but we found that for $\rho_c \gtrsim 10^7$gm/cm^3, the initial deviations from equilibrium of our model gave rise to gravitational collapse! However, one can imagine that the general results obtained would be applicable to more compact configurations which will not collapse for sufficiently small oscillations. We conclude, however, that large amplitude oscillations would result in collapse for the more compact configurations, at least down to the neutron star stage, as discussed below.

We have assumed that the nuclear energy source is very close to the surface, where for example He^4 might be converting to C^{12}. Using the form $P_n = C_o(\rho - \rho_1)^2$ for the over-pressure in the next-to-last mass zone, we found that $C_o = 9.0 \times 10^{11}$ and $\rho_1 = 5.0 \times 10^4$ give reasonable oscillations with $\rho_2 = 7.0 \times 10^{14}$. Figure 1 shows the final asymptotic time dependence of the outside radius and the central density. The time intervals between the large radius maxima are nearly equal to the fundamental frequency for small pulsations. Note the appearance, however, of the small secondary maxima. In more realistic models, the secondary maxima might well be larger, thus shortening the apparent oscillation period. Many optical variable stars have been noted to have periods shorter than the fundamental.[10]

Similar calculations were also carried out for very large amplitude degenerate neutron star pulsations. For a neutron star of total mass $M = 1.0 \times 10^{33}$ gm, an oscillation period of 1.33 sec can be obtained if the maximum radius is taken to be 2600 km. In Figure 2 we have graphed the outside radius as a function of time. Note that the period agrees with the estimates of Hoyle and Narlikar[11]. However, even if their mechanism to avoid high neutrino energy loss at maximum contraction is possible, we find that the density at maximum extension is so low that free neutron decay takes place again resulting in neutrino loss. In fact, the damping time for the pulsations is of the order of a few months.

Radio Emission

In this section we discuss plasma mechanisms for generating the observed radio emissions. We assume that the basic energizing mechanism is an electric field in a layer where the scale height is smaller than the mean free path of thermal electrons. Thus we have a "low density" plasma in which ordinary MHD is invalid, and the electric field may have a component parallel to the magnetic field[12]. From Faraday's law, $-\dot{B} = c\nabla \times E$, we see that maximum E is obtained for maximum $|\dot{B}|$, which corresponds to maximum $|d\rho/dt|$ in the deeper layers of the star, where MHD is valid. We will see that fields $B \gtrsim 60$ gauss are required, and thus the magnetic pressure dominates the plasma pressure in the low density layer. Changes in B in the interior will then be communicated to the atmosphere. But max $|d\rho/dt|$ occurs twice during a pulsation cycle, and thus maximum E occurs twice during a cycle, making the stellar pulsation period twice the radio period.

The appearance of an electric field E with a component parallel to the magnetic field will then give rise to longitudinal plasma oscillations in which the electrons move parallel to B, at the plasma frequency ω_p. These oscillations themselves do not radiate energy since there is no magnetic field associated with them, but in an inhomogeneous medium there is coupling with other radiative plasma wave modes[13], and the emitted radiation would then be at the plasma frequency ω_p. Kundu and Chitre[14] have also mentioned the possibility of plasma oscillations as an exciting mechanism.

Let us discuss radiation at 100 Mc. The corresponding plasma frequency is $2\pi \times 10^8$ sec^{-1}, and the electron density is then $n = 1.24 \times 10^8$ cm^{-3}. Let us assume that the thermal temperature is 10^4 °K, and that the local density scale height is $l_0 = 6 \times 10^3$ cm, a typical scale height for a white dwarf atmosphere. The electron mean free path λ_e is then[15], with $\ln \Lambda = 15$ (where $\ln \Lambda$ is the Coulomb logarithm),

$$\lambda_e = 1.3 \times 10^5 T^2/n \ln \Lambda = 7.2 \times 10^3 \text{ cm.}$$

Thus, $\lambda_e > l_0$, and MHD breaks down. The l_0 assumed here may actually be too small, since the intense radio output from the pulsar will probably distend the atmosphere. This same effect, however, will also raise the temperature, and we will still have $\lambda_e > l_0$.

We assume that the average power emitted from this layer is about 10^{28} erg/sec.[16] The conversion from longitudinal oscillations to radiative plasma waves could occur in many (say 10^7) bursts per fundamental pulsation, so that if W is the energy stored in the plasma oscillations before release, $W \sim 10^{21}$ ergs. The number N of participating electrons is $N = 4\pi R_0^2 l_0 n$, and if $R_0 = 2.5 \times 10^8$ cm, $N \simeq 6 \times 10^{29}$. Thus each electron must possess, on the average, a energy w of 1.6×10^{-9} ergs. But if δr_{cs} is the charge separation associated with the oscillations and E is the electric field strength, we have from $E = en\delta r_{cs}$, $w = eE\delta r_{cs} = E^2/n$. Therefore $E = 0.42$ esu.

We may now use Faraday's law to find the time rate of change of the magnetic field by assuming various values for the scale δr_E of the electric field. One possibility is to set $\delta r_E \simeq l_o$, but for a large-scale magnetic field, possibly dipole-like, we might also set $\delta r_E \simeq R_o$. In any case, $\dot{B} \simeq cE/\delta r_E$, and for $\delta r_E = 6 \times 10^3$ cm, $\dot{B} \simeq 2 \times 10^6$ gauss/sec, which implies a rather large field if $\dot{B} \simeq B$, for a radio pulsar oscillation period of about one second. However, for $\delta r_E = 2 \times 10^8$, $\dot{B} \simeq 60$ gauss/sec, which is more reasonable. Note that if our estimate of l_o is too low, then smaller values of $\dot{B}$ will suffice to produce the emission.

It is observed that the pulses have characteristically short rise times. We note that mode coupling from the plasma oscillations to radiative plasma waves is non-linear and therefore short groups of bursts with rather sharply defined frequency spectra might be expected to occur.

Gyroradiation might also be a contributing source. For 100 Mc, the appropriate field strength is 36 gauss, but for non-relativistic electrons, the decay time for the electron kinetic energy[17] in such a field is roughly 5×10^5 seconds. Thus, while it might be possible to get enough energy stored as electron kinetic energy, many more orders of magnitude of participating electrons would be required for the necessary emissivity. However, gyroradiation can still contribute since fast rise-time bursts might result from stimulated "maser" emission[18] operating in layers where the electron distribution is non-thermal. This would give nearly monochromatic emission, the band-width being determined by inhomogeneities in the magnetic field and by the electron relativity parameter. If the pulsation amplitude is large, the two radio bursts per stellar oscillation may have different characteristics. On the other hand, it is likely that the amplitudes are small since the calculations described in the previous section show that collapse occurs when the amplitudes are too large.

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FOOTNOTES

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Table 1. WHITE DWARF MODELS WITH DEGENERATE ELECTRON PRESSURE ONLY

cgs units

Log Density (gm/cc)	Mass/10^{33}	Radius/10^{8}	Period
10	2.81	1.31	1.64
9.5	2.78	1.81	2.07
9	2.68	2.46	2.63
8.5	2.54	3.28	3.40
8	2.32	4.30	4.49
7.5	2.00	5.55	6.11
7	1.60	7.04	8.67
6.5	1.17	8.81	13.0
6	.785	10.9	20.5

Table 2. WHITE DWARF MODELS WITH HAMADA-SALPETER CORRECTIONS

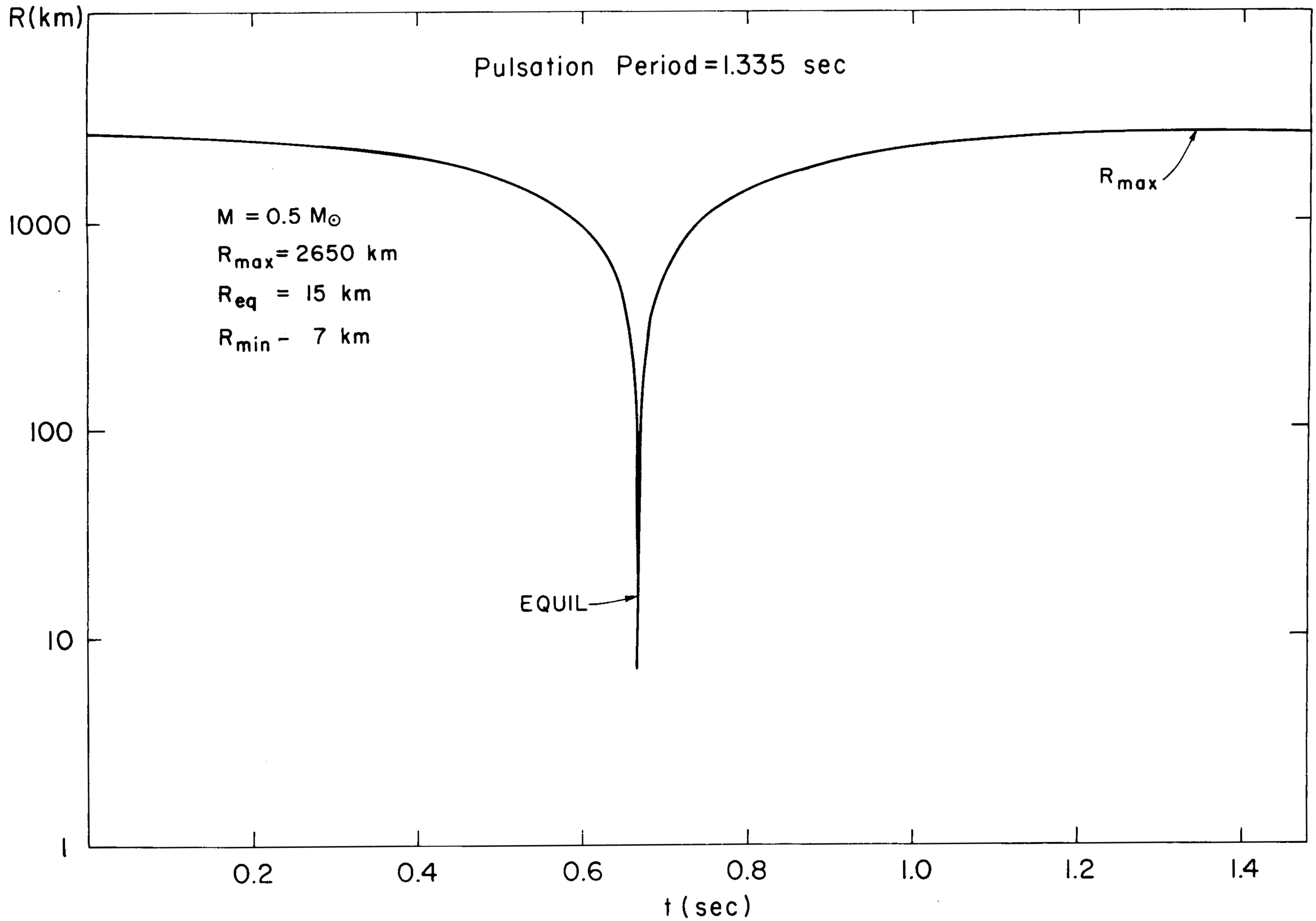
cgs units

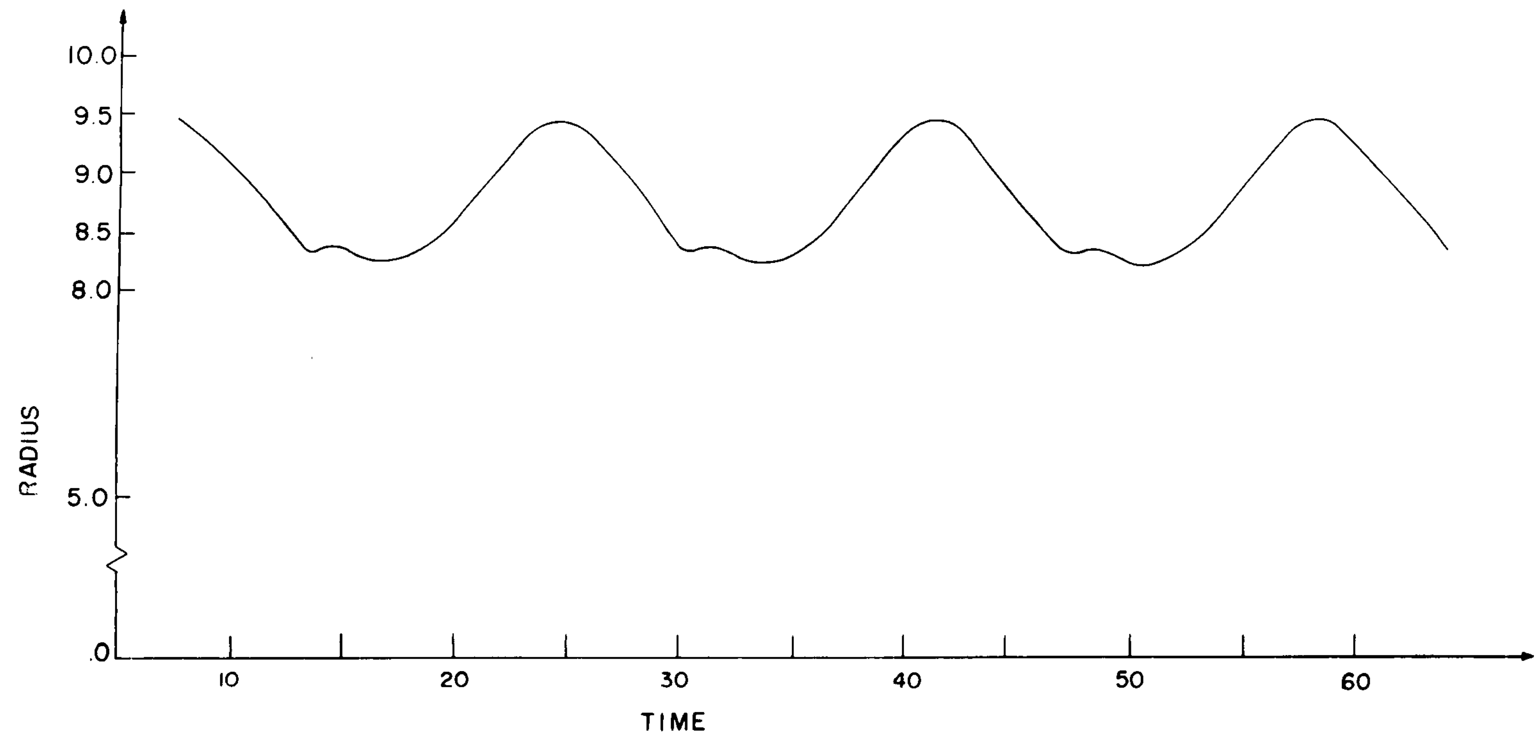
Log Density (gm/cc)	Mass/10^{33}	Radius/10^{8}	Period
10	2.75	1.30	1.64
9.5	2.71	1.79	2.06
9	2.63	2.43	2.61
8.5	2.48	3.24	3.37
8	2.26	4.24	4.44
7.5	1.94	5.45	6.03
7	1.54	6.89	8.52
6.5	1.12	8.57	12.7
6	.743	10.5	19.9

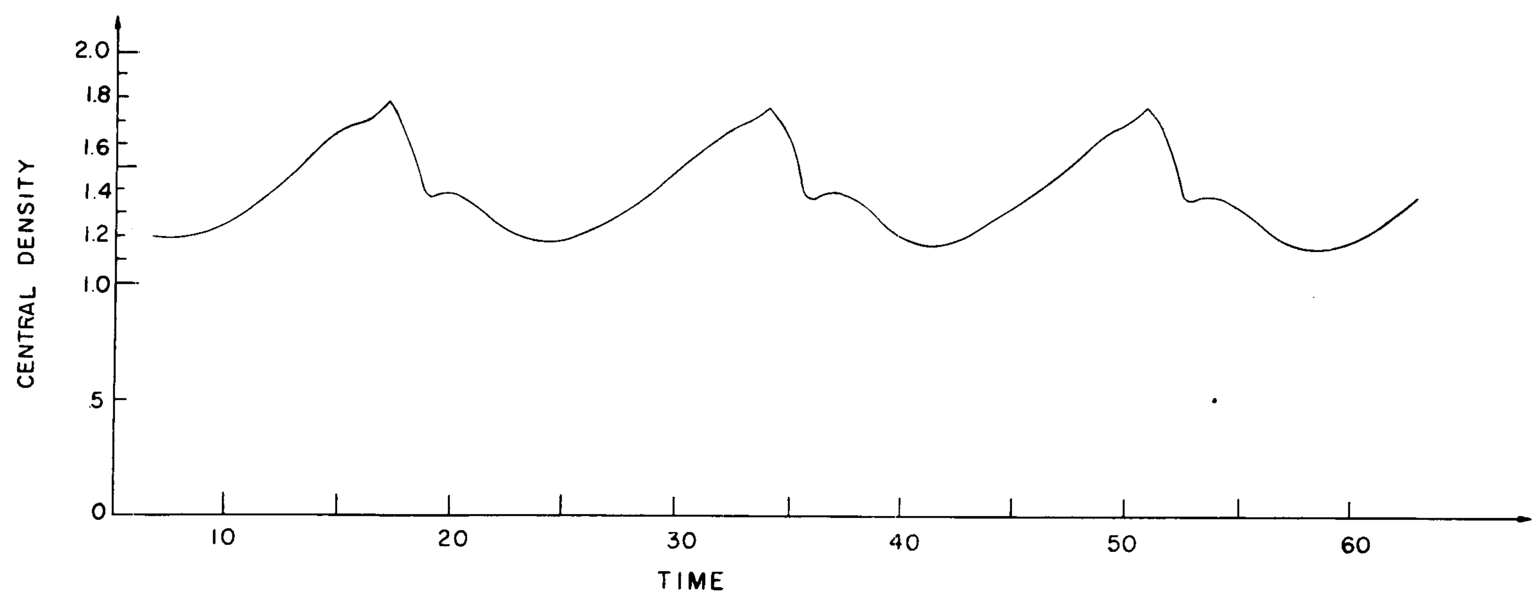
FIGURE CAPTIONS

Fig. 1. Outer radius (in units of 100 km) and central density of a white dwarf (in units of 10^6 gm/cc) vs. time (in sec.). $M = 8.38 \times 10^{32}$ g.

Fig. 2. Outer radius (in km.) of a neutron star as a function of time (in sec.).


R(km)
Pulsation Period = 1.335 sec
M = 0.5 M⊙
R max = 2650 km
R eq = 15 km
R min - 7 km
R max
EQUIL
1000
100
10
1
0.2
0.4
0.6
0.8
1.0
1.2
1.4
t (sec)


RADIUS
10.0
9.5
9.0
8.5
8.0
5.0
.0
10
20
30
40
50
60
TIME


CENTRAL DENSITY
2.0
1.8
1.6
1.4
1.2
1.0
.5
0
10
20
30
40
50
60
TIME